ELECTRIC FIELD IN THE TRANSVERSE CROSS SECTION OF THE CHANNEL OF AN MHD GENERATOR

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During the motion of a partially ionized gas in magnetohydrodynamic channels the distribution of the electrical conductivity is usually inhomogeneous due to the cooling of the plasma near the electrode walls. In Hall-type MHD generators with electrodes short-circuited in the transverse cross section of the channel the development of inhomogeneities results in a decrease of the efficiency of the MHD converter [1]. A two-dimensional electric field develops in the transverse section. Numerical computations of this effect for channels of rectangular cross section have been done in [2, 3]. At the same time it is advisable to construct analytic solutions of model problems on the potential distribution in Hall channels, which would permit a qualitative analysis of the effect of the inhomogeneous conductivity on local and integral characteristics of the generators. In the present work an exact solution of the transverse two-dimensional problem is given for the case of a channel with elliptical cross section stretched along the magnetic field. The parametric model of the distribution of the electrical conductivity of boundary layer type has been used for obtaining the solution. The dependences of the electric field and the current and also of the integral electrical characteristics of the generator on the inhomogeneity parameters are analyzed.

1. We consider some general relations for cylindrical Hall-type MHD channels with an arbitrary shape of the cross section S. We shall assume that in the channel $|x| < L$, $(y, z) \in S$ there is a stationary flow of an anisotropically conducting gas. We shall take the applied magnetic field to be homogeneous

$$\mathbf{B} = -B\mathbf{e}_z,\ B = \text{const} > 0$$

and the parameter of magnetohydrodynamic interaction and the magnetic Reynolds number to be small compared to unity.

We also assume that the lateral surface of the channel is made up of a large number of thin closed electrodes separated from each other by thin dielectric fillings [2], so that the boundary Γ of an arbitrary section x = const can be taken as equipotential. The external load is connected to the end sections of the channel $x = \pm L$.

It is assumed that the length of the channel is much larger than its characteristic transverse dimension. In this case the conductivity is assumed to be a known function of the transverse coordinates $\sigma = \sigma(y, z)$. The Hall parameter of the electrons β is taken to be constant and given, and the Hall parameter of ions is assumed to be negligibly small.

A homogeneous profile is given for the velocity of the flow:

$$\mathbf{V} = U\mathbf{e}_x,\ U = \text{const} > 0.$$

In the presence of a turbulent boundary layer the real velocity profile will be exaggerated. In this case the integral electrical characteristics of the generator operating with a weakly ionized plasma are more sensitive to sudden changes of the conductivity in the temperature boundary layer than to the inhomogeneities of the velocity field.

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Disregarding the effect of the end zones, we arrive at the problem of two-dimensional distributions of the electric field **E** (y, z) and the current density **j**(y, z) with the subsequent computation of the integral electrical characteristics of the generator. The system of equations to be used is of the form

$$\mathrm{rot}\,\mathbf{E} = 0, \qquad \mathrm{div}\,\mathbf{j} = 0$$
$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{V}\times\mathbf{B}) - \mathbf{j}\times\boldsymbol{\beta}, \qquad \boldsymbol{\beta} = \beta\mathbf{B}/B \;. \tag{1.1}$$

We introduce the electric potential χ, such that $\mathbf{E} = -\nabla\chi$. Since the electric field is independent of the coordinate, it has the form

$$\chi(x, y, z) = E_{\parallel}\, x + \varphi(y, z), \quad E_{\parallel} = -E_x = \mathrm{const} \geqslant 0 \;.$$

The constant $E_{\parallel}$ occurs in the solution of the boundary value problem for the function φ as a parameter and is later determined through the value of the external load from Ohm's law for a complete circuit. From (1.1) we conclude that the function φ(y, z) satisfies the following elliptic equation:

$$\frac{\partial^2\varphi}{\partial y^2} + (1+\beta^2)\frac{\partial^2\varphi}{\partial z^2} + \frac{\partial\ln\sigma}{\partial y}\frac{\partial\varphi}{\partial y} + (1+\beta^2)\frac{\partial\ln\sigma}{\partial z}\frac{\partial\varphi}{\partial z} = E_*\frac{\partial\ln\sigma}{\partial y}$$
$$E_* = UB + \beta E_{\parallel} = \mathrm{const} \tag{1.2}$$

For Eq. (1.2) in region S, it is necessary to solve a Dirichlet problem with zero boundary conditions at the contour

$$\varphi|_{\Gamma} = 0 \;. \tag{1.3}$$

It is easy to see that the solution of the formulated problem has the form

$$\varphi(y, z) = E_*\varphi_1(y, z)$$

wherein the function φ_1(y, z) does not contain the quantity E_*. If the solution of the boundary value problem is known, then all the integral characteristics of the generator can be computed with its use.

Let us consider the relationship of the integral characteristics and the quantities characterizing the distribution of the conductivity, field, and current in the transverse cross section of the channel. We introduce the following quantity:

$$\sigma_e = \langle j_y\rangle / E_* \;.$$

Here and below the angular brackets denote averaging of the function over the section S. In the case $\sigma \equiv$ const the quantity σ_e coincides with the effective conductivity [4]. Making use of the expression for $\langle j_y\rangle$ obtained by averaging Ohm's law (1.1), we get

$$\sigma_e = (1+\beta^2)^{-1}\langle\sigma(1+f)\rangle \qquad (f = E_y / E_* = -\partial\varphi_1/\partial y) \;. \tag{1.4}$$

The function f(y, z) does not contain E_* as a parameter. In the case of homogeneous conductivity, $f \equiv 0$, since the boundary value problem (1.2), (1.3) for σ = const has only a trivial solution. It follows from the boundary condition (1.3) that the electrical power generated by the transverse field $\mathbf{E}_\perp = E_y\mathbf{e}_y + E_z\mathbf{e}_z$ on the currents, when averaged over the cross section, is equal to zero

$$\langle\mathbf{j}_\perp\mathbf{E}_\perp\rangle = -S^{-1}\iint_{S} \mathbf{j}_\perp\nabla\varphi\, dS = -S^{-1}\iint_{S}\mathrm{div}(\varphi\mathbf{j}_\perp)\, dS = -S^{-1}\int_{\Gamma}\varphi\mathbf{n}\mathbf{j}_\perp\, d\Gamma = 0 \;. \tag{1.5}$$

On the other hand, the computation of the quantity $\langle\mathbf{j}_\perp\mathbf{E}_\perp\rangle$ with the use of Ohm's law yields the expression

$$\langle\mathbf{j}_\perp\mathbf{E}_\perp\rangle = (1+\beta^2)^{-1}\{\langle\sigma[E_y^2 + (1+\beta^2)E_z^2]\rangle + \langle\sigma f\rangle E_*^2\} \;.$$

From this equation and from (1.5) it follows that $\langle\sigma f\rangle \leq 0$. Then from (1.4) we arrive at the inequality

$$\sigma_e \leqslant \langle\sigma\rangle / (1+\beta^2) \;. \tag{1.6}$$

The equality in (1.6) is satisfied only for $\sigma \equiv$ const.

Using Ohm's law again, we compute the mean density of the longitudinal current j_x:

$$\langle j_x \rangle = \sigma_e \beta UB + (\beta^2 \sigma_e - \langle \sigma \rangle) E_\parallel \ .$$

In the idling mode we will have $\langle j_x \rangle = 0$, and $E_\parallel$ attains its maximum (compared to the operating modes of the generator) value E_m, where

$$E_m = \beta UB \sigma_e (\langle \sigma \rangle - \beta^2 \sigma_e)^{-1} \ . \tag{1.7}$$

On connecting an external resistance R we will have $E_\parallel = kE_m$. In the mode of electrical power generation for the load coefficient k, we obtain

$$0 < k = R / (R + r) < 1 \ .$$

Here r is the equivalent internal resistance of the generator. For a usable electrical power N, electrical efficient η, Joule dissipation Q, and r, we obtain the following equations:

$$\begin{gathered} N = 2LS \langle j_x \rangle E_\parallel = 2LSU^2 B^2 \alpha \beta^2 \sigma_e k (1-k), \ \alpha = E_m / \beta UB \\ \eta = \alpha \beta^2 k (1-k) / (\alpha \beta^2 k + 1), \ Q = 2LSU^2 B^2 \sigma_e (\alpha \beta^2 k^2 + 1) \\ r = 2LS^{-1} (\langle \sigma \rangle - \beta^2 \sigma_e)^{-1} \end{gathered} \tag{1.8}$$

Here $\alpha \leq 1$ is the shunting coefficient of Hall emf. The maximum value of η for variation over k is attained at the following value of the load coefficient:

$$k = k_m = \alpha^{-1} \beta^{-2} [(1 + \alpha \beta^2)^{1/2} - 1]$$

and is equal to $\eta_m = 1 - 2k_m$.

It follows from equations (1.7), (1.8) and inequality (1.5) that for fixed values of L, S, U, B, β, and k in the presence of inhomogeneous distribution of the conductivity, N, η, Q, E_m, and r decrease in comparison with their values for $\sigma = \langle \sigma \rangle \equiv \text{const}$.

Thus, in order to determine the integral characteristics of the generator, it is necessary to know the value of σ_e, which can be calculated only from the known solution of the boundary value problem.

2. We now pass on to dimensionless variables

$$y_* = \sqrt{1+\beta^2}\, y / a, \quad z_* = z / a, \quad \varphi_* = \sqrt{1+\beta^2}\, \varphi / E_* a \tag{2.1}$$

in Eq. (1.2).

Here a is the characteristic transverse dimension of the channel. In the deformed plane $y_* z_*$ the function φ_* is described by an equation with isotropic differential operator

$$\frac{\partial^2 \varphi_*}{\partial y_*^2} + \frac{\partial^2 \varphi_*}{\partial z_*^2} + \frac{\partial \ln \sigma}{\partial y_*} \frac{\partial \varphi_*}{\partial y_*} + \frac{\partial \ln \sigma}{\partial z_*} \frac{\partial \varphi_*}{\partial z_*} = \frac{\partial \ln \sigma}{\partial y_*} \ . \tag{2.2}$$

In the plane $y_* z_*$ we introduce the polar coordinates

$$\rho = \sqrt{y_*^2 + z_*^2}, \quad \tan \theta = y_* / z_* \ . \tag{2.3}$$

A quite simple solution of the Dirichlet problem for Eq. (2.2) can be constructed if under transformation (2.1) region S goes over into a circle of unit radius and the electrical conductivity depends only on ρ. In this case the transverse cross section of the MHD channel is an ellipse, whose major semiaxis a is directed along the vector B and the minor semiaxis is equal to $a/\sqrt{1+\beta^2}$. The lines $\sigma = \text{const}$ form a family of ellipses similar to the boundary contour Γ.

With these assumptions, Eq. (2.2), written in variables (ρ, θ), becomes

$$\frac{\partial^2 \varphi_*}{\partial \rho^2} + \frac{1}{\rho} \left(1 + \frac{d \ln \sigma}{d \ln \rho}\right) \frac{\partial \varphi_*}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \varphi_*}{\partial \theta^2} = \frac{1}{\rho} \frac{d \ln \sigma}{d \ln \rho} \sin \theta \ . \tag{2.4}$$

We shall seek the solution of the homogeneous Dirichlet problem for Eq. (2.4) in the form

$$\varphi_* = \Phi(\rho) \sin \theta \qquad (0 \leqslant \rho \leqslant 1, \ 0 \leqslant \theta \leqslant 2\pi) \ .$$

For the function $\Phi(\rho)$ we have the following boundary value problem:

$$\Phi'' + \rho^{-1}(1 + d\ \ln\sigma / d\ln\rho)\Phi' - \rho^2\Phi = \rho^{-1} d\ln\sigma / d\ln\rho$$
$$\Phi(0) = \Phi(1) = 0. \tag{2.5}$$

The vanishing boundary condition for Φ at $\rho = 0$ follows from the requirement of continuity of the potential at the coordinate origin. In the case of an arbitrary dependence $\sigma(\rho)$ the solution of problem (2.5) cannot be written in terms of tabulated functions. The solution can be obtained in the final form if, for example, the dependence $d\ln\sigma/d\ln\rho$ is approximated by a piecewise constant function. Let us consider the following model law of variation of the conductivity:

$$\sigma = \sigma_0 = \text{const} \quad (0 \leqslant \rho < \rho_*), \qquad \sigma = \sigma_0(\rho_* / \rho)^\varkappa \quad (\rho_* \leqslant \rho \leqslant 1). \tag{2.6}$$

The quantity ρ_* defines the dimensionless radius of the zone of homogeneous conductivity. For $\varkappa > 0$ the dependence in (2.6) simulates the law of decrease of the electrical conductivity in the boundary layer. Below we shall take $\varkappa > 0$, although the solution obtained below can be investigated also for negative values of $\varkappa$.

Using the dependence (2.6), we obtain the boundary value problem for the equation with discontinuous coefficients

$$\Phi'' + \rho^{-1}[1 - \varkappa H(\rho - \rho_*)]\Phi' - \rho^{-2}\Phi = -\varkappa\rho^{-1}H(\rho - \rho_*)$$
$$\Phi(0) = \Phi(1) = 0. \tag{2.7}$$

Here $H(\rho - \rho_*)$ is the Heaviside unit function. The general solution of Eq. (2.7) contains two arbitrary constants in the region $0 < \rho < \rho_*$ and two in the region $\rho_* < \rho < 1$. For $\rho = \rho_*$ the conditions of equality of the functions $\Phi(\rho)$ and $\Phi'(\rho)$ are imposed, which follow from physical conditions of continuity of the potential and the normal component of the current at the boundary of the homogeneous zone. These requirements, together with the boundary conditions, determined the unique continuously differentiable solution of the problem (2.7), which has the form

$$\Phi(\rho) = [1 - G(\lambda_1 - \lambda_2)\rho_*^{-1}]\rho \qquad (0 \leqslant \rho \leqslant \rho_*)$$
$$\Phi(\rho) = \rho - G[(1 - \lambda_2)(\rho/\rho_*)^{\lambda_1} + (\lambda_1 - 1)(\rho/\rho_*)^{\lambda_2}] \qquad (\rho_* \leqslant \rho \leqslant 1) \tag{2.8}$$
$$\lambda_{1,2} = \frac{\varkappa}{2} \pm \sqrt{1 + \frac{\varkappa^2}{4}}, \quad G = [(\lambda_1 - 1)\rho_*^{-\lambda_2} + (1 - \lambda_2)\rho_*^{-\lambda_1}]^{-1}$$

The second derivative Φ'' is discontinuous at $\rho = \rho_*$, which corresponds to the discontinuity of the space charge density and is related to the choice of the distribution of the conductivity (2.6), which is not smooth at $\rho = \rho_*$.

For the components of the vectors $\mathbf{E}$ and $\mathbf{j}$ we obtain the following equations:

$$E_x = -E_\parallel = \text{const}$$
$$E_y = -E_*(\rho^{-1}\Phi\cos^2\theta + \Phi'\sin^2\theta)$$
$$E_z = (1 + \beta^2)^{-1/2}E_*(\rho^{-1}\Phi - \Phi')\sin\theta\cos\theta \tag{2.9}$$
$$j_x = -E_\parallel\ \sigma(\rho) + (1 + \beta^2)^{-1}\beta E_*\sigma(\rho)(1 - \rho^{-1}\Phi\cos^2\theta - \Phi'\sin^2\theta)$$
$$j_y = (1 + \beta^2)^{-1}{}_*E_*\sigma(\rho)(1 - \rho^{-1}\Phi\cos^2\theta - \Phi'\sin^2\theta)$$
$$j_z = (1 + \beta^2)^{-1/2}E_*\sigma(\rho)(\rho^{-1}\Phi - \Phi')\sin\theta\cos\theta$$

We note that equation (2.9) remains valid even in the case of arbitrary dependence $\sigma(\rho)$. The function $\Phi(\rho)$ must be a solution of the problem (2.5) corresponding to the chosen function $\sigma(\rho)$.

3. We now turn to the analysis of the obtained solution. The function $\Phi(\rho)$ has a single extremum at the point $\rho = \rho_0$, which is determined from the solution of the transcendental equation

$$(\lambda_1 - 1)\rho_*^{1-\lambda_2} + (1 - \lambda_2)\rho_*^{1-\lambda_1} = (\lambda_1 + 1)\xi^{\lambda_1 - 1} - (\lambda_2 + 1)\xi^{\lambda_2 - 1}$$
$$(\xi = \rho_0 / \rho_*).$$

In the interval $0 < \rho < \rho_*$, $\Phi(\rho)$ is linear; therefore, we always have $\rho_0 > \rho_*$, i.e., the points of extremum of the potential $\varphi_*(\rho, \theta)$ lie in the region of the boundary layer. A similar result was obtained in the numerical computations in [2, 3]. A direct calculation shows that $\partial\xi / \partial\rho_* < 0$; therefore, with the decrease of the thickness of the boundary layer the extremums shift toward the boundary of the homogeneous zone $\rho = \rho_*$. For a fixed value of ρ_* the quantity ρ_0 increases with the parameter $\varkappa > 0$.

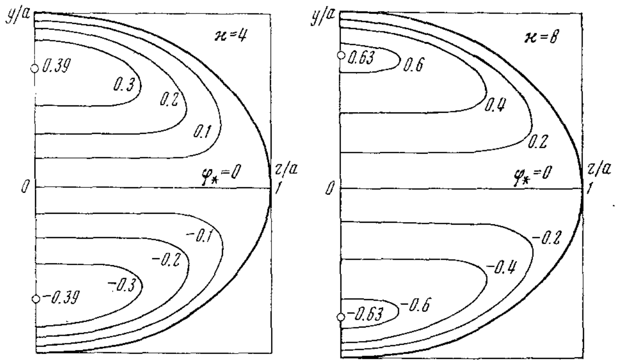

Fig. 1

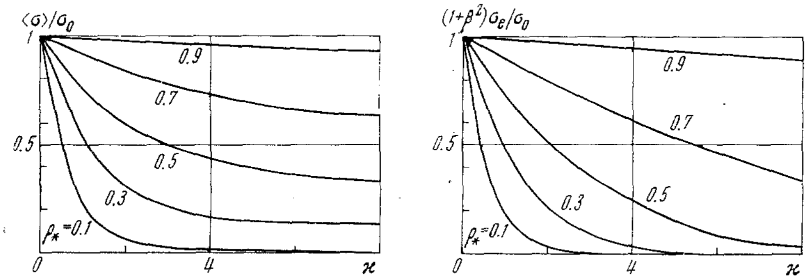

Fig. 2

It is not difficult to verify that in its main body $\Phi'(\rho)$ is positive. Hence the value $\rho = \rho_0$ is the point of maximum of the function $\Phi(\rho)$. Thus, at $\rho = \rho_0$, $\theta = \frac{1}{2}\pi$ a maximum occurs, while at $\rho = \rho_0$, $\theta = \frac{3}{2}\pi$ we get a minimum of the nondimensional potential $\varphi_* = \Phi(\rho)\sin\theta$.

An increase of the thickness of the boundary layer leads to a growth of the function $\Phi(\rho)$. For a constant radius of the homogeneous zone ρ_* and for large values of $\varkappa$ we obtain the asymptotic equations

$$\Phi \approx \left(1 - \frac{\varkappa\rho_*^{-1}}{\varkappa - 1 + \rho_*^{-\varkappa}}\right)\rho \qquad (0 \leqslant \rho \leqslant \rho_*) \tag{3.1}$$

$$\Phi \approx \rho - \frac{\varkappa - 1 + (\rho/\rho_*)^{\varkappa}}{\varkappa - 1 + \rho_*^{-\varkappa}} \qquad (\rho_* \leqslant \rho \leqslant 1) .$$

Equations (3.1) show that for $\varkappa \to \infty$ the limiting distribution of $\Phi(\rho)$ is discontinuous:

$$\lim_{\varkappa\to\infty} \Phi(\rho) = \rho \quad (0 \leqslant \rho < 1), \qquad \lim_{\varkappa\to\infty} \Phi(1) = 0 .$$

This behavior of $\Phi(\rho)$ is caused by the fact that for $\varkappa \to \infty$ the quantity σ in the boundary layer tends to zero. As a result, in any region $\rho < 1 - \varepsilon$ away from the wall the current density tends to zero and the vector $\mathbf{E}$ tends to the induced field $\mathbf{E}_i = -\mathbf{V} \times \mathbf{B}$, whose potential at the wall is nonzero.

The level lines of the dimensionless potential φ_* in the first and fourth quadrants of the dimensionless physical plane y/a, z/a are shown in Fig. 1 for $\beta = 1$, $\rho_* = 0.5$, and for $\varkappa = 4$ and 8.

The component E_y of the electric field is alternating; in the homogeneous zone, $E_y < 0$. The equation of the line 1, on which E_y vanishes, has the following form in the coordinates (ρ, θ):

$$\operatorname{cotan}^2 \theta_l = -\rho\Phi'/\Phi . \tag{3.2}$$

This last equation has a real solution $\theta_l(\rho)$ in the region $\rho_0 \leq \rho \leq 1$, where ρ_0 is a maximum of $\Phi(\rho)$. In the deformed plane $y_* z_*$ the line l represents a closed curve symmetric with respect to the coordinate axes tangent to the circle $\rho = 1$ at $\theta = 0$ and π and to the circle $\rho = \rho_0$ for $\theta = \frac{1}{2}\pi$ and $\frac{3}{2}\pi$. The maximum value of E_y along the coordinates is obtained at the boundary contour at the points $(\rho = 1, \theta = \frac{1}{2}\pi)$ and $(\rho = 1, \theta = \frac{3}{2}\pi)$.

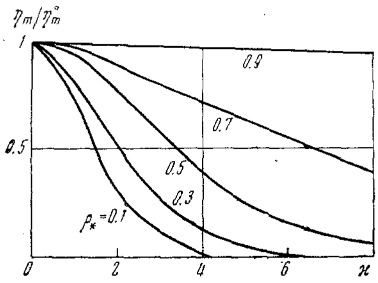

Fig. 3

The component E_z of the field is equal to zero in the homogeneous zone. In the boundary layer, E_z is positive in the first and the third quadrants and negative in the second and fourth. The maximum and minimum values of E_z are attained at the boundary $\rho = 1$ on the rays $\theta = \frac{1}{4}\pi, \frac{5}{4}\pi$, and $\frac{3}{4}\pi, \frac{7}{4}\pi$, respectively.

The component j_y of the current density is everywhere positive and lies in the following ranges:

$$j_1(\rho) \leqslant j_y(\rho, \theta) \leqslant j_2(\rho)$$
$$j_1(\rho) = (1+\beta^2)^{-1} E_*\sigma(1-\rho^{-1}\Phi), \quad j_2(\rho) = (1+\beta^2)^{-1} E_*\sigma(1-\Phi') \tag{3.3}$$

The component j_y is constant in the core and falls off with the increase of ρ in the boundary layer. The maximum of j_y is attained at the points of the contour Γ for $\theta = \frac{1}{2}\pi, \frac{3}{2}\pi$ and is equal to $j_2(1)$. At these points the retarding Lorentz force is maximum, and they are the most hazardous in possible rupture of the viscous boundary layer [5]. With the increase of the parameter $\varkappa$ or with a decrease of ρ_* the component j_y decreases.

The qualitative characteristics of the behavior of the current component j_z depend on the values of $\varkappa$. In the core, $j_z \equiv 0$, while in the boundary layer, $|j_z|$ increases monotonically with the increase of ρ in the range $0 < \varkappa \leq \varkappa_0$. For $\varkappa > \varkappa_0$ the maximum of $|j_z|$ along ρ appears at $\rho = \rho_+$. The values of $\varkappa_0$ are the roots of the transcendental equation

$$\left[\frac{1+\lambda_1(\varkappa_0)}{1+\lambda_2(\varkappa_0)}\right]^{1/[\lambda_1(\varkappa_0)-\lambda_2(\varkappa_0)]} = \rho_*^{-1} .$$

For ρ_+ we have the following equation:

$$\rho_+ = \rho_*\,[(1+\lambda_1)/(1+\lambda_2)]^{1/(\lambda_1-\lambda_2)} .$$

The maximum of j_z in the first quadrant of region S is attained at the boundary at the point ($\rho = 1$, $\theta = \frac{1}{4}\pi$), if $0 < \varkappa < \varkappa_0$, and inside the region at the point ($\rho = \rho_+$, $\theta = \frac{1}{4}\pi$), if $\varkappa > \varkappa_0$. For $\varkappa \to \infty$ the absolute maximum of j_z gets shifted to the boundary of the homogeneous zone.

A computation of σ_e corresponding to the obtained solution yields the following equation:

$$\sigma_e = \frac{\sigma_0}{1+\beta^2}\,\frac{(\lambda_1+1)\rho_*^{\lambda_2} - (\lambda_2+1)\rho_*^{\lambda_1}}{(\lambda_1-1)\rho_*^{-\lambda_2} + (1-\lambda_2)\rho_*^{-\lambda_1}} . \tag{3.4}$$

For the integral-mean conductivity corresponding to the distribution (2.6), we obtain

$$\langle\sigma\rangle = \sigma_0\left[\rho_*^2 + \frac{2}{2-\varkappa}(\rho_*^{\varkappa} - \rho_*^2)\right] \qquad (\varkappa \neq 2) \tag{3.5}$$
$$\langle\sigma\rangle = \sigma_0\rho_*^2(1 - 2\ln\rho_*) \qquad (\varkappa = 2) .$$

For $\varkappa \to \infty$, σ_e tends to zero, while $\langle\sigma\rangle$ tends to a finite limit $\sigma_0\rho_*^2$. The dependences of $\langle\sigma\rangle / \sigma_0$ and $(1+\beta^2)\sigma_e/\sigma_0$ on the power exponent $\varkappa$ in the law of decrease of the conductivity (2.6) are shown in Fig. 2. The dimensionless radius of the homogeneous zone ρ_* serves as a parameter of this family of curves.

The replacement of σ_e by the quantity $\langle\sigma\rangle / (1+\beta^2)$ in the computation of the integral characteristics of the generator leads to appreciable errors at sufficiently large values of $\varkappa$ or sufficiently small values of ρ_*. The dependences of the ratio η_m/η_m^0 on the exponent $\varkappa$ are shown in Fig. 3 for $\beta = 2$. Here η_m is the maximum possible electrical efficiency of the generator computed from the two-dimensional theory, and η_m^0 is the maximum efficiency computed under the assumption $\sigma \equiv \langle\sigma\rangle = \text{const}$. The parameter of the family of curves is ρ_*.

Let us now investigate briefly the distribution of that part of the local electric power, which is due to the components of the vector **E** in the plane of the transverse cross section of the channel. For the quantity $n = \mathbf{j}_\perp \mathbf{E}_\perp$, we obtain the expression

$$n(\rho, \theta) = (1+\beta^2)^{-1}E_*^2\sigma(\rho)[\rho^{-1}\Phi(\rho^{-1}\Phi - 1)\cos^2\theta + \Phi'(\Phi' - 1)\sin^2\theta] . \tag{3.6}$$

The function $n(\rho, \theta)$ is alternating, since it does not give any contribution to the integral power [see Eq. (1.5)]. The condition $n = 0$ determines the line along which the coordinates ρ and θ are connected by the relation

$$\cot an^2\theta_\gamma = -\Phi'(\Phi' - 1) / [\rho^{-1}\Phi(\rho^{-1}\Phi - 1)] \cdot \qquad (3.7)$$

A real solution $\theta_\gamma(\rho)$ of Eq. (3.7) exists in the region $\rho_0 \leq \rho \leq 1$. The line γ is symmetric with respect to the coordinate axes and is a closed curve tangent to the circle $\rho = 1$ at $\theta = 0, \pi$, and to the circle $\rho = \rho_0$ at $\theta = \frac{1}{2}\pi, \frac{3}{2}\pi$. Compared to the line l on which $E_y = 0$, curve γ lies closer to the homogeneous zone: on l, $n = \sigma E_z^2 \geq 0$.

In view of the fact that the transverse current $\mathbf{j}_\perp$ is closed through the electrode wall, the range of positive values of n performs the function of a load in the circuit of the transverse current. The central region bounded by the contour γ plays the role of current source in this circuit.

The generalization of the solution given above to the case of a generator with a frame-type channel, when the transverse section has the previous elliptical shape and the closed electrode frames are inclined to the axis of the channel at an angle δ, does not present any essential difficulties. For this case also the solution is constructed by the method of separation of variables, wherein for the function $\Phi(\rho)$ Eq. (2.5) must be solved with the boundary conditions:

$$\Phi(0) = 0, \Phi(1) = -(E_\parallel / E_*) \cot an\, \delta.$$

The solution obtained above can be used for the improvement of the hydraulic model describing quasi-one-dimensional flow in profiled Hall channels with appropriate geometry of the transverse cross section. Models of this type are usually used in engineering computations of MHD devices [5].

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